ISSN 1022-8594
JUJS

Vol. 44 No. 1, pp.1-14

The Effect of Bronchial Compliance along Parallel Channel under Constant Flow

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Abstract

In this study, a numerical simulation is carried out to investigate the nature of flow convection along lower and narrower airways of the parallel channel under constant flow rate through the human lung. This paper aim at to examine the effect of compliance with a real scale model channel (18th generation) for constant flow. We have graphically illustrated how the variation of compliance affects every flow convection. We also found that the lung model of parallel networks exhibits a larger effect of capacitance when the capacitive time constant is decreased. This implies that a little change in the capacitance C value causes more flow to be diverted into the capacitive tube, which has an impact on our lung model.

Keywords: Capacitive flow, Compliance, Lumped-model, Oscillatory flow, Parallel Channel.

Introduction

The lung is a vital organ in the respiratory system that participates in gas exchange between O_2 and CO_2 and transport through the airways. Every moment fluid divides in a bifurcated channel. The study of biological flow along multi-bifurcated lung airways is a challenging work undoubtedly. Only in the most straightforwardly constructed circumstances is it possible to map the velocity field, solve the flow field, and show how the flow field and atmospheric pressure are related. In biology fluid flow problems are typically complex and rarely have a simple solution. The successful application of the lumped model that is apparent in medical science to have microscopic behavior of lung mechanics was studied by Zamir[1]. A model with lumped parameters has been used for macroanalysis. By using an analogy of an RC circuit and a lumped parameter model, Otis et al. [2] showed the mechanical

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behavior of the lungs. When the time-constants of the various paths are different, they demonstrated that the breathing frequency is a crucial factor in the distribution of ventilation. Ultman et al. [3] and High et al. [4] transformed the RC model into the RLC model by adding the inertia effect for high frequency. They investigated the pendelluft flows at tidal volumes of 5 to 15 mL and frequencies, $f = 6\ to\ 26$ Hz during ventilation. According to their findings, asymmetries in compliance and inertance created more pendelluft than asymmetries in resistance. Measurements are used to estimate compliance. Woolcock et al. [5] reported that dynamic compliance depends on frequency. They contrasted dynamic compliance with static compliance. They also noted that anomalous breathing was seen when dynamic compliance was substantially lower than static value.A human lung is like a branch of a tree from the trachea (0^{th} generation) to the alveolar end (23^{rd} generation). The airflow along the tree seems to be a simple mutual motion due to inspirations and expirations. So, flow simulation is done by numerical calculations with suitable modeling where in-vivo inspection of airflows is almost impossible. Moreover, the treatment strategy varies depending on the physical condition of the patient. Along with the human lung, HFOV regulates the steady oscillatory flow, and flows in the lung channel are affected by compliance and resistance. Numerous experimental and numerical studies using the HFOV technique have been conducted on respiratory flow. Elad et al. [6] have created a nonlinear lumped model for asymmetric connections based on structural and physiological characteristics. They arrived at modified time-dependent expressions of a compartment's resistance and compliance. They also demonstrated how the flow distribution in daughter tubes may be impacted by the unequal compliance of respiratory airways.In contrast to straight tubes, Tanaka et al. [7] found that the HFOV approach uses a secondary flow to enable effective diffusion in curved and bifurcated tubes. As additional extensions of the study of Elad et al. [6], a non-linear lumped parameter model for asymmetric bronchial bifurcations has been calculated. Hirahara et al. investigated the numerical analysis of airflow along lung channels with asymmetric compliance using numerical and experimental methods[8]. They demonstrated, using the lung model, that the flow for inhomogeneous compliance ratio results in irreversible flow.Experimental analysis on pendelluft flow was investigated and the effect of pendelluft in gas exchange was discussed without the importance of inertia effect by Lee et al.[9] and Kim et al.[10]. To numerically simulate the patterns of respiratory flow in human lungs, Calay et al. [11] and Inagaki et al. [12] collaborated. This simulation demands that more investigation is needed to study on compliance effect. Paul Hou et al.[13] studied with a methodology of complex electrical model for the respiratory system.Ahmmed and Sultana [14] have looked into the impact of inertia for lumped parameter models with constant flow. They demonstrated that it takes longer to reach steady state at larger inertial forces.The authors infered that the effect of compliance on tube during respiratory flow will be burning issue of research in future.

In the present study, an incompressible viscous flow along the realistic model channel of the human lung is considered and numerical simulation is performed after the analytical solution of the circuit model equation. Though the fluid flow model and electrical model are not on the same track, we just attempt to perform an analogy study. This analogy study outlined that the effect of compliance exists in a respiratory channel and affects the resistive flow, inductive flow, and capacitive flow even if very small in range.

Anatomy of Real Lung

In the human lung, there are primarily two states: the gas stream state and the gas exchange or diffusion state. The lung tree has 23 generations (G0 to G23, where G stands for generation) depending on the respiratory function and anatomical configuration. From the terminal bronchioles until the beginning of the lung (G0), gas moves without exchanging (G16). The anatomical dead space is the volume between G0 and G16. Gas exchange takes place primarily in the remaining volume, which is known as the diffusion-dominated state and spans from G17 to G23. Gas exchange occurs in the presence of respiratory bronchioles, alveolar ducts, and alveolar sacs.The adult lung contains roughly 2.5 to 3 liters of air out of 5 liters in several branched networks of 2^{N-16}(N for the total number of generation) tubes. For the numerical experiment, we chose the generations G18 to G20 because we anticipate vigorous gas mixing and exchange in the bottom zone of the lung. Since the presence of laminar flow in this area is crucial for understanding how exchange and transport mechanisms operate.

Model Channel of G18 to G20

The lung channels are naturally flexible. It is essentially impossible to do laboratory experiments using real lungs due to experimental constraints. To protect the local deformation of the channel wall in this case, we suggest a stiff model channel. In order to have realistic flow characteristics for our model channel, the experimental value of lung compliance is included in the numerical experiment (see Ahmmed,[15]). The entire experiment also takes into account the geometric arrangement and dynamical behavior. Fig.1 shows a doubly-bifurcated 2D respiratory model channel for adult human lungs, from G18 to G20. The primary feature of our model channel is the symmetric bifurcation of one tube into two smaller tubes (see Ahmmed [14]). The mother tube's axis and the fabrication process ensured that each branch of the channel was axially symmetric (G18). By modifying the refractive index, the test channel became rectangular.

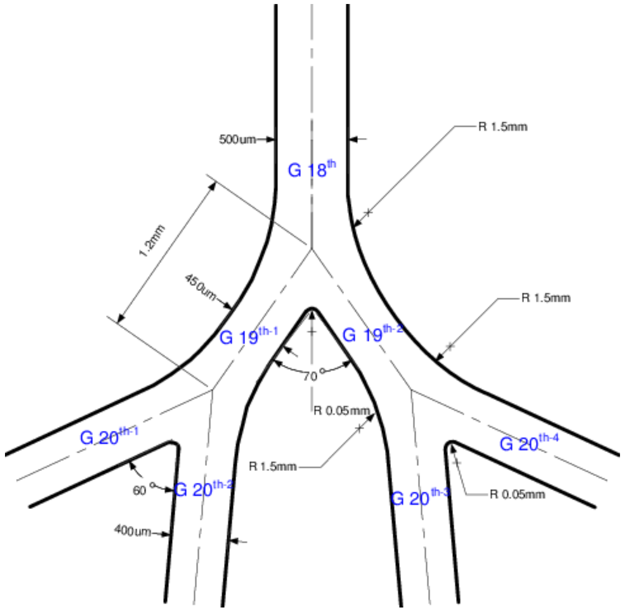

Figure1. Dimension of the test channel

The ratio of our test channel's diameters $D_{18} : D_{19} : D_{20} \equiv 10:9:8$ demonstrates how the flow channel changed linearly. G18, G19, and G20 each have different channel widths, which are $500\mu m$, $450\mu m$, and $400\mu m$ respectively. The depth of our channel is represented by the average diameter ($500\mu m$) of the rectangular channel. Middle branch G19's length is $1.2mm$ along the center line, and G18 and G20's are long enough to extend up to completely developed flow. G19's scale measurement is taken into consideration since channel resistance is quite sensitive to changes in radius. Real flow phenomena and hence gas exchange processes should therefore be anticipated in this intermediate branch. The dividing point's radius of curvature is $50\mu m$. Upper and lower junction angles are 70 degrees and 60 degrees, respectively.

The outer and inner branches of G20 are bent by 65° and 5° about the mother tube, respectively. In consideration of outer G20 and middle G19, the channel can be considered as an alveolated bend wherein the flow mechanism would be interesting to lung investigation.

Modeling of Governing Equation

Factors that affect the pressure flow relation in the bifurcated bronchial channel include the channel's length and width, bifurcation angle, elasticity, curvature at the junction and driving pressure (steady or oscillatory) applied along it. The importance of a lumped model is increased by the flow-through complicated geometry [17]. Three parameters, including resistance (*R*), inertance (*L*), and compliance (*C*) make up our lumped model for simulating flow. The amount of resistance needed to move a unit charge of air through the model channel is measured in pressure. When shear stress is applied on the entire surface area of the tube, the total resistance to flow can be stated in terms of the resistive flow rate q_R as

$$R = \frac{\Delta p}{q_R} \tag{1}$$

The acceleration or deceleration of the flow field causes inertia. A fluid's inertance (*L*) is the amount of pressure difference needed to gradually change the inductive flow rate (q_L) with time.

$$\Delta p = L\frac{dq_L}{dt} \tag{2}$$

The elastic or viscoelastic tube is used to simulate airflow in a human lung's bronchial channel. When air flows along a bronchial tube, the whole volume of the tube may change so there is an effect known as compliance (in fluid) or capacitance (in an electric). We consider a flexible balloon-like alveolus of the lung connected at the end of the rigid model channel. Let V is the volume of the balloon at the static condition and Δp is driving pressure to inflate the balloon.Then, mathematically compliance *C* is defined as,

$$C = \frac{\Delta v}{\Delta p} \tag{3}$$

We consider the incompressible flow within the model channel.Therefore, the only way to change the balloon's volume is to modify the fluid volume of the balloon. When the flow rate Δq oscillates over a period of time Δt, there is a small change in volume Δv.

$$\Delta q = \frac{\Delta v}{\Delta t} \tag{4}$$

If Δp is a continuous function of time, Δq also become a function of time. Then we get from Eqns. (3) & (4),

$$\Delta q = c\,\frac{dp}{dt} \tag{5}$$

In presence of compliance, the pressure-flow for an instantaneous time interval t would be defined by

$$p_1 = \int_0^t \Delta p_1 dt = \frac{1}{C}\int_0^t \Delta p_1 dt \tag{6}$$

As shown in Fig.1, where G19L and G19R are parallel in the 19th generation and G20LL, G20LR, G20RL, and G20RR are parallel in the 20th generation, we now assume that the lumped parameters *R*, *L*, and *C* are in parallel when there is a driving pressure drop(Δp).

The parallel system's overall flow rate is provided

$$q = q_R + q_L + q_C = \frac{\Delta p}{R} + \frac{1}{L}\int \Delta p dt + C\,\frac{d(\Delta p)}{dt} \tag{7}$$

Where, respectively, the flow rate, resistance, inertance, and compliance of the i-th channel are q_i, R_i, L_i, and C_i, and $\Delta p = p_2 - p_1$ is causing pressure at the channel's intake. We determine the overall flow rate by assuming a constant pressure drop in the parallel channel of the *RLC* system.

$$q = q_R + q_L + q_C = \frac{\Delta p}{R} + \frac{1}{L}\int \Delta p dt + C\,\frac{d(\Delta p)}{dt} \tag{8}$$

We differentiate with respect to time using the inertial time constant, $t_L (= L / R)$ and the capacitive time constant, $t_C (= RC)$.

$$t_C\,\frac{d^2(\Delta p)}{dt^2} + \frac{d(\Delta p)}{dt} + \frac{1}{t_L}\Delta p = R\,\frac{dp}{dt} \tag{9}$$

Analytical Solution of Model Equation

To calculate the capacitive effect of the compliant tube, the inertial effect of the inductive tube, and the viscous effect of the resistive tube while preserving the total flow rate constant, the Eqn. (9) reduces to

$$t_C \frac{d^2(\Delta p)}{dt^2} + \frac{d(\Delta p)}{dt} + \frac{1}{t_L}\Delta p = 0 \tag{10}$$

Its resolution is determined by the type of the associated equation's roots

$$t_C \lambda^2 + \lambda + \frac{1}{t_L} = 0 \tag{11}$$

Generally, the roots are given by

$$\lambda = \frac{-1 \pm \sqrt{1-(4\,t_C/t_L)}}{2t_C}$$

For underdamped flow condition, λ is complex for $4t_C > t_L$. If λ_1 and λ_2 are two complex (conjugate) roots, then the solution of Eqn. (10) is

$$\Delta p(t) = e^{ct}\left\{C_1 \cos(dt) + C_2 \sin(dt)\right\} \tag{12}$$

Where C_1 and C_2 are to be calculated as arbitrary constants.

Resistive, inductive, and capacitive flow rates display non-dimensional changes that are $\bar{q}_{R,L,C}(t) = \dfrac{q_{R,L,C}(t)}{q}$ such that

$$\bar{q}_R(t) = e^{ct}\{\overline{C_1}\cos(dt) + \overline{C_2}\sin(dt)\} \tag{13}$$

$$\bar{q}_L(t) = \frac{e^{ct}}{2}[\overline{C_1}\{-\cos(dt) + 2dt_C \sin(dt)\} + \overline{C_2}\{-\sin(dt) - 2dt_C \cos(dt)\}] + 1 \tag{14}$$

$$\bar{q}_C(t) = \frac{e^{ct}}{2}[\overline{C_1}\{-\cos(dt) - 2dt_C \sin(dt)\} + \overline{C_2}\{-\sin(dt) + 2dt_C \cos(dt)\}] \tag{15}$$

Additionally, the pressure drop can be expressed non-dimensionally as

$$\overline{\Delta p}(t) = \Delta p / Rq = e^{ct}\left\{\overline{C_1}\cos(dt) + \overline{C_2}\sin(dt)\right\} \tag{16}$$

where, $\overline{C_1} = C_1 / Rq$ and $\overline{C_2} = C_2 / Rq$

Initially, $t = 0$ Eqns. (13) reduce to

$$\bar{q}_R(0) = \overline{C_1} \tag{17}$$

$$\bar{q}_L(0) = -\overline{C_1}/2 - \overline{C_2}dt_C + 1 \tag{18}$$

$$\bar{q}_C(0) = -\overline{C_1}/2 + \overline{C_2}dt_C + 1 \tag{19}$$

Because there are three starting flow rates and two unknown constants, we can only recommend prescribing two initial flow rates. We must study the dynamic system of our problem for t_L and t_C and such that the entire inflow q flows through the resistive tube since the sum of the inductive and capacitive flow rates is 0.

$$\bar{q}_R(0) = 1, \bar{q}_L(0) + \bar{q}_C(0) = 0 \tag{20}$$

$$\bar{q}_R(0) = 1, \bar{q}_L(0) = 0, \bar{q}_C(0) = 0 \tag{21}$$

$$\overline{C_1} = 1, \quad \overline{C_2} = 1/(2dt_C) \tag{22}$$

The initial conditions are

which give, after some algebra

The values in Eqns.(22) satisfy the conditions $\overline{C_1} = 1$ in Eqns. (17) and $\bar{q}(0) = \bar{q}_R(0) + \bar{q}_L(0) + \bar{q}_C(0) = 1$ in Eqns.(8). The non-dimensional flow rates in Eqns. (13)-(15) can be displayed as functions of time with the aid of values for $\overline{C_1}$ and $\overline{C_2}$. For the process to be finished, the time constant values are needed.

Results and Discussion

In the present study, we evaluate the flow behavior together with the human lung using an electrical analogue of parallel flow under underdamped situations.To attain quantitative results of fluid dynamical problems using the electrical analogy is a very difficult assignment. We start first by the second-order governing equation as in Eqn. (9) and solve for the underdamped condition with a constant flow rate. In underdamped flow, the resistive flow rate (q_R), the inertial flow rate (q_L), the capacitive flow rate (q_C) are all taken into consideration.The inertial time constant (t_L) and capacitive time constant (t_C), respectively, are used to calculate the intensity of flow through the inductive tube and capacitive tube.The values $L = 1.44 \times 10^7 [Pa.s^2/m^3]$, $R = 1.44 \times 10^7 [Pa.s/m^3]$ and $C = 10.5 \times 10^{-8} [m^3/Pa]$ are obtained from Hirahara et al. [8].In order to finish the procedure graphically, the values of are needed. Under a constant flow rate, q into the system, flow rates in a resistive tube, q_R an inductive tube, q_L and a capacitive tube, q_C are parallel. The system's total flows are normalized to a value of one. Initially, q_L and q_C are both set to zero, giving them a starting value of 1 of q_R. Therefore, the system's original flow rate is carried into the system by the resistive tube.

Then the numerical simulation is continued for $t_L = 1$sec and $t_C = 1.5$sec. As seen in Fig. 2, the non-dimensional flow rates in Eqns. (13)–(15) are represented as functions of time. In this instance, the inertial flow (green dash-dot curve) increases quickly from 0 to the whole flow into the system and then q_C , q_R decreases to zero as a result. The capacitive flow rate (dotted curve, blue) decreases to zero at $q = 0$ for $t_C = 1.5$ sec, whereas the inductive flow rate approaches steady state at $q = 1$ for $t_L = 1$sec. Additionally, the system's resistive flow rate (solid curve, black) decreases to zero. Oscillations connected to under-damping accompany with this airflow simulation through the lung.

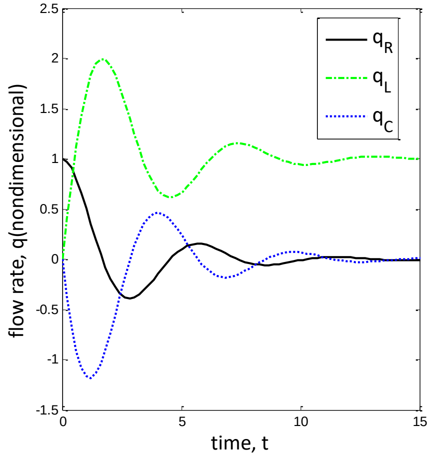

Figure 2. RLC system in Parallel under constant flow rate(underdamped case)

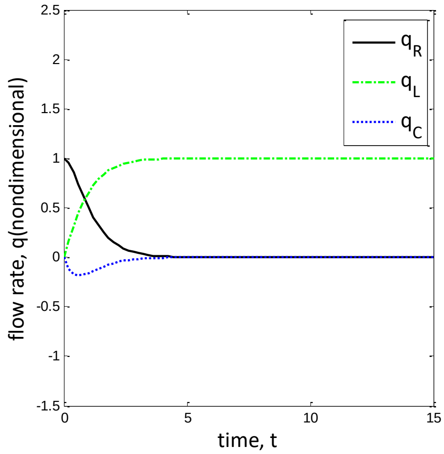

Figure 3. Flow rates q_R, q_L and q_C based on R, L and $C = C/5$.

The capacitive flow rate curve shows the system's compliance (*C*) effects for a minimum time constant, t_L =1 s and $t_C = 1.5$ s as in Fig.2. According to the graph, the capacitive flow rate starts out zero and eventually falls to zero, but the capacitor inflates before reaching steady-state zero. This indicates that due to a relatively short time constant, the capacitor absorbs fluid mass before reaching its final value of zero.

The values of compliance (C), for which the influence of compliance is visible in the flow rates q_R, q_L and, q_C are modified while the values of resistance (R) and inertance (*L*) are left unchanged as experimental values.In Fig.3, the smaller value $t_C(C/5)$ is taken into account to determine the system's capacitor's state. Therefore, $t_L = 1$ sec is constant throughout the experiment and changes depending on the compliance value (C).Here, we can observe that the resistive flow is gradually decreasing and stopping after a short period of time since the momentum is finite and cannot be generated by any external force, and the flow is being resisted by the alveolar compliance acting as a recoil force. The inductive flux increases quickly and rapidly achieves a steady state. Here, a lower compliance (C) value diverts more flow into the capacitive tube and achieves stability.

From Fig.2 and Fig.3,We can reach the conclusion that the lung model of parallel networks exhibits a stronger effect of capacitance the shorter the capacitive time constant.

Due to high resistance and inertial force, it is difficult to evaluate the influence of compliance in the biological field, particularly in the 18^{th} to 20^{th}generations of the human lung. When the total flow is constant in the parallel system, it is also necessary to analyze the effect of compliance in each resistive, capacitive, and inductive tube separately.

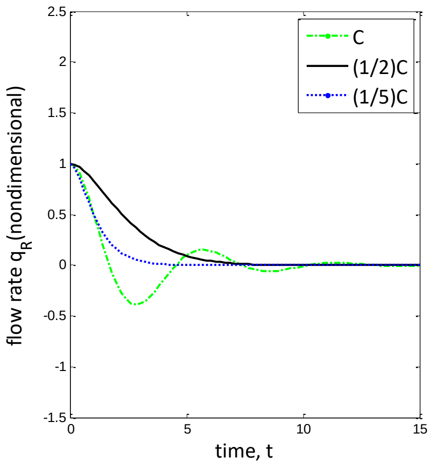

Figure 4. Variations in resistive flow q_R with changes of t_C .

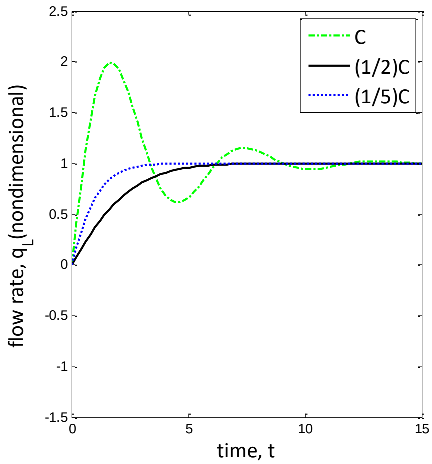

Figure 5. Variations in inertial flow, q_L with the change of t_C .

Fig.4 represents the fluctuation of q_R while $t_C = f(iC), i \in (0, \infty)$ under constant flow as in Eqn. (13) for various values of C . We can see that when the compliance force decreases, the gradient of the resistive flow gradually increases from a negative value. It implies that a smaller compliance force minimizes the resistive flow rate atsteady state more quickly.

According to Eqn. 14, Fig.5 illustrates the fluctuation of q_L for various values of C under constant flow rate. With the reduction of compliance force, we observe that the gradient of inertial flow is occasionally dropping and occasionally increasing before reaching steady-state. It implies that, up until a specific level of the value, the gradient of inertial flow reduces, and that, after the value is reached, the gradient of inertial flow increases.

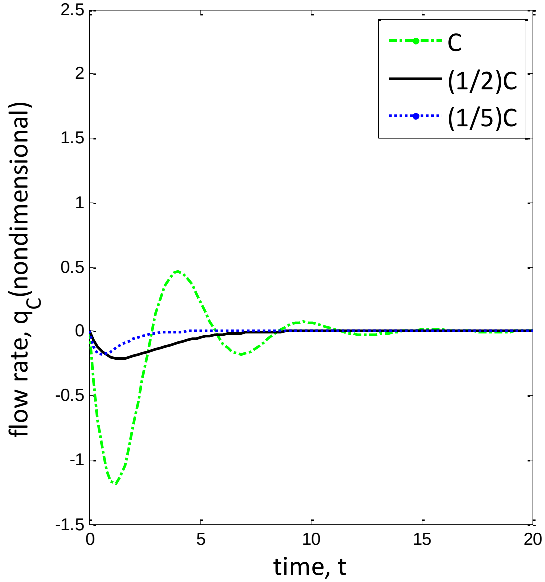

Figure 6. Variation in inertial flow, q_C with changes t_C .

The change q_C for various values of C under a constant flow rate is shown in Fig.6. This effect can be calculated mathematically from Eqn.(15). The flow curve's gradient is initially negative before turning positive and reaching zero at steady state. The variations q_C show that the fluid particles are received by the compliant tube and are discharged within a very short capacitive time constant. A lower value of the capacitance (C) causes more flow to be diverted into the capacitive tube, which stabilizes the dynamics of our model channel. Higher capacitance values again have the opposite effect.

Conclusion

Respiration is a biological system where most of the respiratory channels are irregularly shaped by nature. We considered a lumped parameter model through parallel channel of human lung with constant flow for incompressible air. The RLC system's natural properties are represented by its free dynamics. Any change in the values R, L, C has the potential to affect the system's intrinsic properties, which show the system's dynamic behavior, and could result in respiratory disease or clinical intervention. For the entire experiment described in this work, the under-damped condition is used. We demonstrated that the resistive and capacitive flows are ultimately eliminated when the inertial flow embraces the entire flow into the system. For the variation of compliance force, the dynamics of resistive flow, capacitive flow, and inertial flow are investigated. We discovered that lower compliance force requires longer for higher inertial flow to reach steady state ($q(t)=1$) and shorter for lower resistive flow to reach steady state ($q(t)=0$).

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