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A Computational Model for Age-Structured Population Projection of Bangladesh

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Abstract

In this paper, we develop a computational model for age-structured population projections of Bangladesh on the basis of the present population trend based on data from BBS and ICDDR,B. We project the first age-group population by least squares method and the other age-group population by our computational model for which we estimate the death rates of each age-group also by least squares methods. Validation of the model is reported and the computed results are compared with other projections.

Keywords: Age-structured Population Projection, Computational Model, Death rates, Least Squares method.

1. Introduction

All over the world population study is a crucial field. The world population rises more swiftly over the twentieth century than even before in human history, grew from 1.6 billion in 1900 to 6.1 billion in 2000 (United Nations, 2001a). Population study has especial impact on developing country like Bangladesh. According to BBS census 2011 around 1015 persons live per square kilometers and is considered the eighth most populous country in the world [1]. Over population is one of the most burning issues for Bangladesh.

Information about the size and distribution of future population is extremely valuable. By these information policy-makers can built a suitable future plan for the government regarding socio-economic, socio-cultural, socio-demographic demands of the growing population of a nation. In order to make an effectual planning for the demands of different age-group it is very important to predict the age-structured population of the country. For example, if we can project the population of 0-4 year's children, we can provide necessary medical care and baby food to reduce the mortality rate and keep children healthy and nourished. We can afford

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necessary educational support for 4-15 year's children, to educate the people of our country. Also for the reduction of unemployment problem, human resource development programs can be delivered for young people [2]. Medical facilities can be developed for aging and elderly-aging people and etc.

Bangladesh achieved position on the world map as an independent nation on December 16, 1971 following the victory at the war of Liberation from 25 March to December 16, 1971 from Pakistan after a tremendous sacrifice. At the time of Bangladesh's independence in 1971 the size of the population was around 70 million.The current population of Bangladesh is 167,160,356 as December, 2018, based on the latest United Nations estimates. Bangladesh population is equivalent to 2.18% of the total world population and the population density is 1278 per square kilometer [3]. The high population density, low economic growth, lack of institutional infrastructure, an intensive dependence on agriculture and agricultural products and various other factors all contribute to make the country week in its economic development and quality of life. Therefore, the age-structured population estimation is a crucial issue in the field of population dynamics. There exist several models to predict age-structured population as follows:

Mathematical Modeling in Population Dynamics was effectively introduced by the British Economist and Demographer Thomas Robert Malthus. In 1798, he published "An Essay on the Principle of Population". Where he argued that Populations grow in size when birth rate exceeds the death rate [5]. Basically, the idea behind the Malthusian Model is the assumption that the dynamics of a single species population can be described by$P'(t) = P(t)f\big(t, P(t)\big)$, where, $P(t)$ =size of the population at time t.The widely used model in Population Dynamics was the Logistic model which was developed by Belgian Mathematician Pierre Francois Verhulst. In 1825 he suggested that the rate of population increase may be limited [7]. This limiting behavior of the population growth is more realistic and described by the following equation: $p'(t) = cp(N - P), p(0) = p_0$, where p represents the total population, t represents the time, p_0 is the initial population and c is a constant representing the relative growth rate and N is new constant is the maximum population size or carrying capacity of population. Age-dependent population models are basic in population dynamics.The first ‘continuous’ population models is incorporating age effects were those of Sharpe-Lotka [9]. Basically, the Sharpe-Lotka [8] model assumes that birth and mortality processes are linear functions of population density which is now known as Von Foerster equation.

The cohort-component method is the most popular and widely used method for producing projections of population at national-level. The cohort-component method divides the launch-year population into age-sex groups(i.e., cohorts) and counts separately for the fertility, mortality and migration behavior of each cohort as it passes through the projection period. In 1895 Cannan introduced the cohort-component method, subsequently used by Bowley (1924). The cohort-component equation is defined for the population at time $(t+n)$ as $P_{t+n} = S[t, t+n] + B[t, t+n] + NM[t, t+n]$. Where, $S[t, t+n]$ is the survived population at time $t+n$, $B[t, t+n]$ is the number of births observed in the period $[t, t+n]$ and $NM[t, t+n]$ is the net migration observed in the period$[t, t+n]$. In 1930 Whelpton developed the procedure for making Cohort-Component population projections. He thought an elaboration in the demographic population balancing equation: $P(t+n) = P(t) + B(t) - D(t) + I(t) - E(t)$. Where, $P(t)$ is the population at time t. $B(t)$ and $D(t)$ are number of births and deaths occurring between t and $t+n$. $I(t)$ and $E(t)$ are the number of immigrants and emigrants from the country during the period t and $t+n$.

In 1949, Leslie used discrete age components giving rise in his famous Age-Structured Leslie Matrices and used both discrete time and age compartments [11]. In 1994, “Evolution in age-structured population”, B. Charlesworth describes that individuals may survive over many years, but reproduction is limited. He formulates this in terms of males and females age-classes and solved by matrix algebra and difference equation of higher order [12]. In [1992], Roos, Diekmann and Metz study the dynamics of populations structured by age, size and any other physiological trait. They introduce a versatile Escalator Boxcar train(EBT)model, which is a numerical method for continuous- time models(De Roos 1988). They apply this model on the size, food and continuous reproduction dependent dynamics [13].

However, in order to investigate the age-structured population projection we propose the following computational model.

2. Age-structuredComputational Model

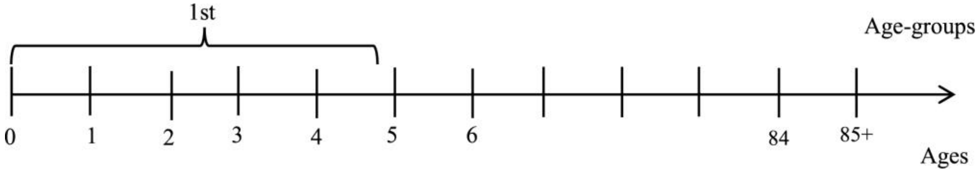
The computational model of age-structured population projection is based on the basis of the following assumptions:

We assume that $u(t,x)$ be the population distribution function with respect to time t and age x.

In this paper, we consider the age of population of Bangladesh from 0 to 85+ years. We split this population (0 to 85+ years) into 18 age-groups i.e. $x_1, x_2, x_3, \cdots\cdots\cdots, x_{18}$. Each age-group contains 5 years interval.

We consider, $u(t_k, x_i)$ represents population of age-group x_i at time t_k, $i = 1, 2, 3, \cdots\cdots\cdots, 18$ denotes age-group and $k = 1, 2, 3, \cdots\cdots\cdots, T$ denotes time and $\mu(t_k, x_i)$ = death rate of age-group x_i at time t_k.

We use age distributed population of 2001 as initial data and the population of the first age-group of the years of 2001 to 2050 as boundary data.



Proposed model:

$$u(t_{k+1}, x_i) = \left(\frac{1}{5}\right) u(t_k, x_{i-1})\big(1 - \mu(t_k, x_{i-1})\big) + \left(\frac{4}{5}\right) u(t_k, x_i)\big(1 - \mu(t_k, x_i)\big) \quad \ldots \ldots \ldots (A)$$

where, $i = 2, 3, \cdots\cdots, 17 \quad and \quad k = 1, 2, \cdots\cdots, T$

And

$$u(t_{k+1}, x_i) = \left(\frac{1}{5}\right) u(t_k, x_{i-1})\big(1 - \mu(t_k, x_{i-1})\big) + u(t_k, x_i)\big(1 - \mu(t_k, x_i)\big) \quad \ldots \ldots \ldots (B)$$

where, $i = 18 \quad and \quad k = 1, 2, \cdots\cdots, T.$

The above two equations represent the transition from one age-group or time period to the next. In our model we consider 0-4, 5-9, 10-14,… … …,80-84, 85+ years people as 1st, 2nd, 3rd,… … …,17th, 18th age-group population respectively. For the computation of age-structured population, first compute the second year of the second age-group population by equation (A). In our computational model we consider twenty percent of first age-group population will be added to eighty percent of second age-group population after one year. Similarly, we can compute the 3rd, 4th, 5th,… …,17th age-group population for the next year and so on. Using equation (B) we compute 18th age-group population by twenty percent of 17th age-group population will be added to 18th age-group population after one year. Because there will be no age-group left to go to the next age-group after one year.

3. Methodology

3.1 Computation of First Age-group Population and Death Rates by Least Squares Method

The method of least squares is one of the most popular methods to determine the best fit line from data. By this method we form a linear relationship between the two quantities time t and first age-group population or death rates p(say). i.e.$p = at + b$, for some constants a and b. So that, we collect some data points $(t_1, p_1), (t_2, p_2), \cdots \cdots \cdots, (t_n, p_n)$, we may define the error associated to saying $p_i = at_i + b, i = 1,2, \cdots \cdots, n$ by

$$e_i = p_i - (at_i + b), i = 1,2, \cdots \cdots, n$$

Now we consider the sum of the squares (Least Squares) of e_i's , i.e.

$$E(a, b) = \sum_{i=1}^{n} {e_i}^2 = \sum_{i=1}^{n} (p_i - (at_i + b))^2$$

To find the values of a and b, the method of least squares consists in minimizing the error, i.e. E such that

$$\frac{\partial E}{\partial a} = 0, \frac{\partial E}{\partial b} = 0 .$$

So,

$$\frac{\partial E}{\partial a} = \sum_{i=1}^{n} 2(p_i - (at_i + b)) \cdot (-t_i) = 0$$

$$\frac{\partial E}{\partial b} = \sum_{i=1}^{n} 2(p_i - (at_i + b)) \cdot (-1) = 0$$

We may rewrite these equations as

$$\left(\sum_{i=1}^{n} {t_i}^2\right) a + \left(\sum_{i=1}^{n} t_i\right) b = \sum_{i=1}^{n} p_i \, t_i$$

$$\left(\sum_{i=1}^{n} t_i\right) a + \left(\sum_{i=1}^{n} 1\right) b = \sum_{i=1}^{n} p_i$$

The values of a and b which minimizes the error must satisfy the following matrix equation:

$$\begin{pmatrix} \sum_{i=1}^{n} {t_i}^2 & \sum_{i=1}^{n} t_i \\ \sum_{i=1}^{n} t_i & \sum_{i=1}^{n} 1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^{n} p_i \, t_i \\ \sum_{i=1}^{n} p_i \end{pmatrix}$$

By solving the above equations we obtained the values of a and b. Now substituting the values of a and b in $p_i = at_i + b, i = 1$, we obtained the 1st age-group population or death rate as shown in figure below:

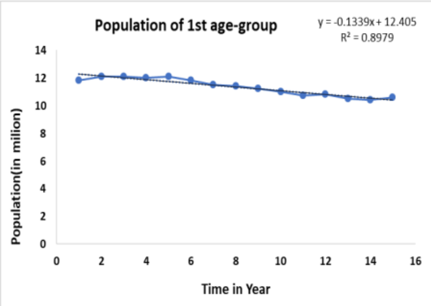


Figure 1: Population of 1st age-group for the years 2001 to2015

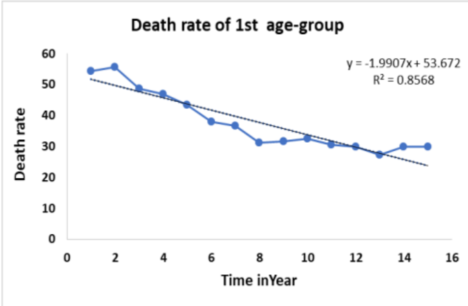


Figure 2: Death rate of 1st age-group for the years 2001 to2015

3.2 Algorithm for the computation of age-structured population

Step 1: We consider the number of year and age. The age distributed population of the year 2001 applies as initial value and the 1st age-group population from 2001 to 2050 applies as boundary value. The age-structured death rate has been considered.

Step 2: Calculation of 1st age-group populations:

For i= 1 & k= 1 : nt+1

$$p(i,k) = at(k) + b$$

Step 3: Calculation of age-structured death rates:

For i= 1 : nx+1 & k= 1 : nt

$$\mu(i,k) = at(k) + b$$

Step 4: Calculation of age-structured populations:

For k= 1 : nt & i= 2 : nx

$$u(i,k+1) = \left(\frac{1}{5}\right)u(i-1,k)\big(1-\mu(i-1,k)\big) + \left(\frac{4}{5}\right)u(i,k)(1-\mu(i,k))$$

And For k= 1 : nt & i=nx+1

$$u(i,k+1) = \left(\frac{1}{5}\right)u(i-1,k)\big(1-\mu(i-1,k)\big) + u(i,k)(1-\mu(i,k))$$

Step 5: Calculation of total population:

For k= 1 : nt+1 & i= 1 : nx+1

$$poput(k) = Sum(u(k,i))$$

4. Results and Discussion

4.1 Model Validity test

Figure shows the comparison of our estimated total population of Bangladesh by Computational model with the population of Bangladesh Bureau of Statitics(BBS) for the years 2001 to 2015.

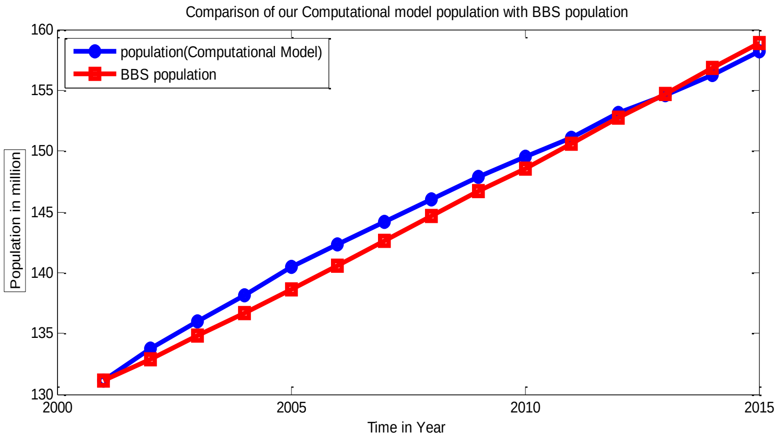


Figure 3: Comparison of Population Projection of Bangladesh by Computational Model with BBS population for the years 2001 to 2015.

$$\%\ Error = \frac{|True\ Value - Experimental\ Value|}{True\ Value} \times 100$$

Year	True Value(BBS)	Experimental Value (Computational model)	% of Error
2001	131.1	131.1	000.0
2002	132.9	133.7	0.6019
2003	134.8	136	0.8902
2004	136.7	138.1	1.0241
2005	138.6	140.4	1.2987
2006	140.6	142.2	1.1379
2007	142.6	144.1	1.0518
2008	144.7	146.1	0.9675
2009	146.7	147.8	0.7498
2010	148.6	149.5	0.6056
2011	150.6	151.1	0.3320
2012	152.7	153.1	0.2619
2013	154.7	154.6	0.0646
2014	156.8	156.2	0.3826
2015	158.9	158.2	0.4405

4.2 Age-Structured Population Projection

Our projected age-structured population of Bangladesh for the years 2001 to 2050 is displayed in Figure 4 to Figure 9. Here we observe that the population up to the 4^{th} age-group (15-19 ages) is decreasing relative to that of the base year 2001. In Figure 6, the population of 7^{th}, 8^{th} and 9^{th} age-group increases gradually. We also observe that the population of 10^{th} -13^{th} age-group increases sharply. This indicates that the movement of the population towards the elder age groups. The population of 14^{th} and 15^{th} age-group increases progressively. From Figure we have also seen that the population of 16^{th} to 18^{th} age-group increases slowly.

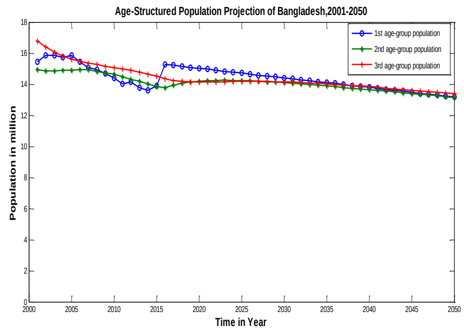


Figure 4: Projections of 1^{st} to 3^{rd} age-group Population for the years 2001 to 2050

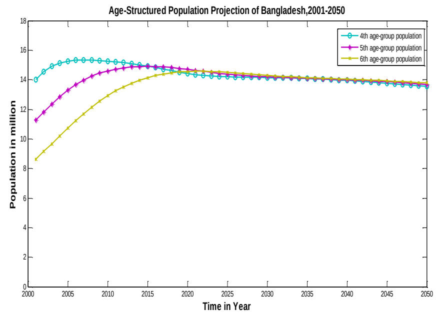


Figure 5: Projections of 4^{tt} to 6^{th} age-group population for the years 2001 to 2050

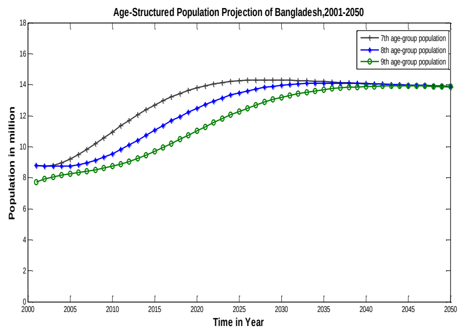


Figure 6: Projections of 7^{th} to 9^{th} age-group Population for the years 2001 to2050

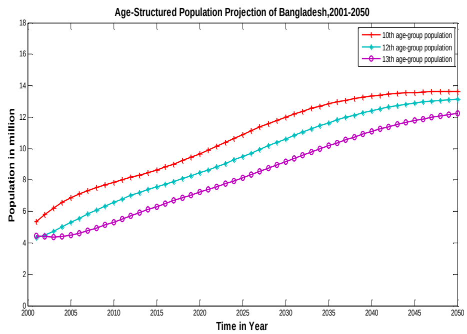


Figure 7: Projections of 10^{th} to 12^{th} age-group population for the years 2001 to2050

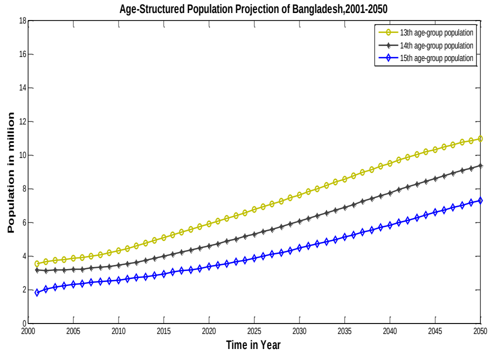


Figure 8: Projections of 13th to 15th age-group Population for the years 2001 to2050

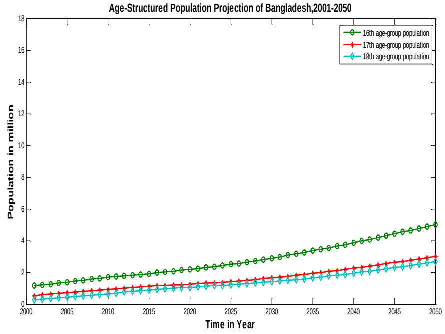


Figure 9: Projections of 16th to 18th age-group population for the years 2001 to2050

4.3 Total Population Projection

Our projected total population marked by "population(Computational model)" for the years 2001 to 2050 are displayed in Figure-10. Here it is clearly observed that, the initial population of our model is 131.1 million for the year 2001 and the predicted population for the year 2050 will be200. 3 million. In the year 2024, the predicted population in our model will be 174.4 million whereas the total population was 76.4 million in 1974. So we can easily conclude that, the total population in 2024 is 2.3 multiple of the total population of 1974.

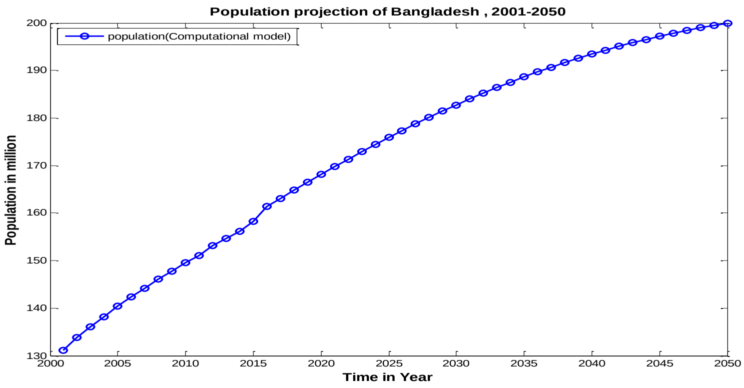


Figure 10: Total Population projection of Bangladesh for the years 2001 to 2050 by Computational Model

4.4 Comparison with other projection

Figure shows the comparison of the predicted population of Bangladesh by our projection for the years 2001 to 2051 with the projection of United Nations population Fund (UNFPA 2014a), EL-Saharty et al. (2014-AFT) and Islam (2000-scenario-III). We have seen that our projected population are very close to the UNFPA (2014a) projection.

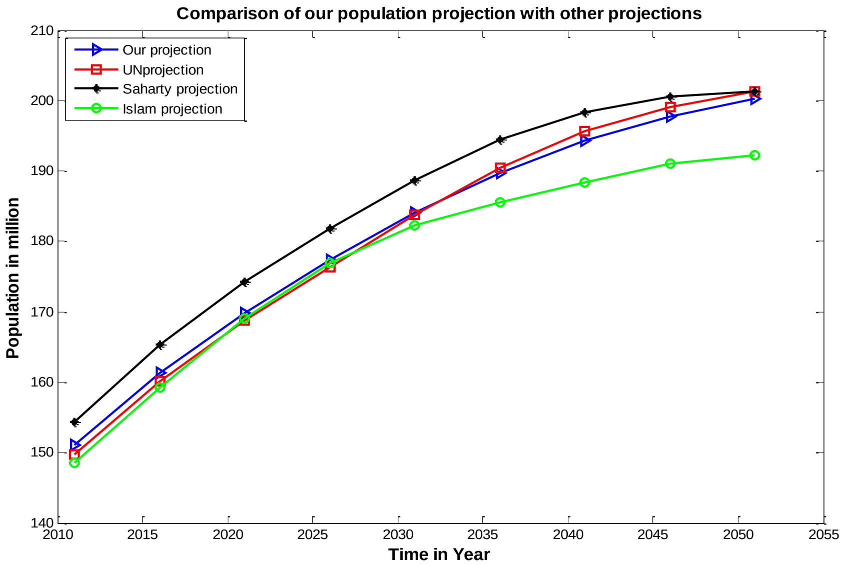


Figure 11: Comparison of Population Projection of Bangladesh by Our Projection with Other Projections

Conclusion

We developed an age-structured computational population model to project the age-specific population. Using Least Squares Curve fitting, we first compute the 1st age-group population and death rates for every age-group based on data(Source: ICDDR,B). We present an algorithm for the computation of the age-structured population for this computational model using data (Source: BBS and ICDDR,B). Age-Structured population projections for the years 2001 to 2050 have been presented. Our results are compared with the real data (BBS) and other recent projections. In order to perform a validity test of our computational model, we compare our results with BBS data for the period 2001-2015, which shows a very good agreement. Our projected results also compare with other projected results for the period 2001-2051.

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