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An Efficient Method for Solving Traveling Salesman Problem

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Abstract

The traveling salesman problem (TSP) consists of a salesman and a set of cities. The salesman has to visit each one of the cities starting from a certain one (e.g. the home city) and returning to the same city. The challenge of the problem is that the traveling salesman wants to minimize the total length of the trip. In this paper, we introduce and analyze a variation of the Traveling Salesman problem. The paper will cover algorithms for TSP to take the decision which optimizes the distance. Our proposed algorithm helps to determine the optimum distance with less number of iteration by easiest way and presents alternative method for solving the classical traveling salesman problem (TSP). Several numerical examples are solved to test the performance of the proposed method. The comparison of the result shows that proposed algorithm is efficient for solving the TSPs.

Keywords : Traveling Salesman Problem, Hungarian Method, Optimal Solution, Computer Algorithm.

1. Introduction

Traveling salesman problem (TSP) is one of the classical demanding combinatorial optimization problems. The objective of the TSP is to minimize the total distance traveled by visiting all the nodes once and only once and then returning to the depot node. An important topic put forward immediately after the transportation problem is the assignment problem. This is particularly important in the theory of decision making. The assignment problem is one of the earliest applications of linear integer programming problem. One of the problems with similar to that an assignment problem is the traveling salesman problem (TSP). Historically the TSP deals with finding the shortest tour in an n-city situation where

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each city is visited exactly once. In this problem, traveling salesman wants to minimize the total distance traveled during his visit of n cities; d_{ij} , $i = 1, 2, ..., n$ is the distance of i th node to j th node. The individual set up distance can be arranged in the form of a square matrix. A TSP is to determine a set of n elements of this matrix, one in each row and one in each column, so as to minimize the sum of elements determined above. Traveling salesman problem is similar to the assignment problem, but here two extra restrictions are imposed. The first restriction is that we cannot select the element in the leading diagonal as we do not follow i again by i. The second restriction is that we do not produce an item again until all the items are produced once. The second restriction means no city is visited twice until the tour of all the cities is completed. The traveling salesman problem (TSP) is one which has commanded much attention of mathematicians and computer scientists specifically because it is so easy to describe and so difficult to solve. The importance of the TSP is that it is representative of a larger class of problems known as combinatorial optimization problems. The TSP problem belongs in the class of such problems known as NP-complete. The TSP is one of the well-known problems for finding an optimal path. A common application of the TSP is the movement of people, equipment and vehicles around tours in aiming to minimize the total traveling distance. A salesman is planning a business trip that takes him to certain cities in which he has customers and then brings him back to the city from where he started. Between some of the pairs of cities that he has to visit, there is a direct air service; while between others there is not such a thing. This type of problem can be easily solved by using the TSP algorithm. Using the TSP algorithm a bank can improve its regular services. Suppose that a bank has many ATM machines. Each day, a courier goes from machine to machine to make collections, gather computer information and service the machines. A problem which may arise in practice at many banks is that in what order should the machines be visited so that the courier's route is the shortest possible? Similarly school-bus routing problem, post routing problem are the application of TSP. The applications of the TSP are not limited to the examples described above. Moreover some mentionable exact and heuristic algorithms have been applied successfully to the TSP by a number of researchers. The importance of the TSP is that it is representative of a larger class of problems known as combinatorial optimization problems. The TSP problem belongs in the class of such problems known as NP-complete. This problem corresponds to finding a shortest Hamiltonian cycle in a complete graph defined by $G = (V, E)$ of n nodes, where $V = \{v_1, v_2 \cdots v_n\}$ denotes the vertices, which represents the nodes of the graph and $E = \{(v_i, v_j) : v_i, v_j \in V, i \neq j\}$ is the set of edges which represents the interconnections between the vertices or nodes. In general, each of the edges is associated with a given distance denoted by d_{ij}. Since we only considered the symmetric TSP, it was assumed that the distance of travelling between two cities is the same in both directions that is $d_{ij} = d_{ji}$.

In the 1800s, the Irish Mathematician W. R. Hamilton and the British Mathematician Thomas Kirkman first defined mathematical problems which were treated related to the Traveling salesman problems. The TSP has been studied intensively and many exact & heuristic numbers of algorithms have been derived. These algorithms include construction algorithms, iterative improvement algorithms, branch-and-bound and branch-and-cut exact algorithms, and many meta-heuristic algorithms. During the 1930s Karl Menger have been first defined the general form of the TSP in Vienna and at Harvard .Karl Menger, who clarifies the problem (TSP), considers the obvious brute-force algorithm and observes the non-optimality of the nearest neighbor heuristic. In the 1950's and 1960's, the problem became very popular in scientific circles in Europe and the U.S.A. In 1954 Dantzig et al. [1] explained the problem as an integer linear program and developed the cutting plane method for its solution. In the next decades, the problem was analyzed by many researchers from mathematics, computer sciences, chemistry, physics, and other sciences. In 1975,Ross and Soland,[2] proposed branch and bound algorithm for the generalized assignment problem. In 1977 Karp [3] approach probabilistic analysis of partitioning algorithms for the traveling salesman problem in the plane. In 1979 Beale, E. M. L., [4] proposed a branch and bound methods for mathematical programming systems. Angluin, D. and Valiant, L. in 1979 [5] proposed a probabilistic algorithm using Hamiltonian circuits and matching's. Balas, E. and Guignard, M. in 1979 [6] introduced a method about branch and bound. Carpaneto, G. and Toth, P. in 1980 [7] proposed a new criteria about branching and bounding for the asymmetric travelling salesman problem. Crowder, H. and Padberg, M.W. in 1980 [8] solve large scale symmetric traveling salesman problems to optimality. In 1981 Balas, E. and Christofides, N., [9] introduced a restricted Lagrangean approach to the traveling salesman problem. In 1982 Olgenant, T. and Jonker, R.[10] proposed a branch and bound algorithm for the symmetric traveling salesman problem based on the 1-tree relaxation. In 1983, Balas, E., McGuire, T. W. and Toth, P.,[11] shows statistical analysis of some traveling salesman algorithms. In 1985 Lawler et al. [12] introduced computational solutions approach for TSP applications. In 2005, Rosenkrantz et al. [13] proposed some of the popular tour construction procedures by the nearest neighbor procedure. In 2006, Applegate et al. [14] derived a detailed review of the applications of the TSP. In 2011, Shaikh Tajuddin Nizami et al. [15] introduced Genetic Algorithm of artificial intelligence for finding the distance of assignment problems. The branch exchange is perhaps most well known iterative improvement algorithm for the TSP. Even today, branch & bound algorithm remains the key ingredient in the most successful approaches for finding high-quality tours and is widely used to generate initial solutions for other algorithms. Branch and bound algorithms are widely used to solve the TSPs. Several authors have proposed (B&B) algorithms based on assignment problem (AP) relaxation of the original TSP formulation. In this paper, we introduce a new method for solving the traveling salesman problem (TSP) and develop a new algorithm for that purpose. In our practical life the TSP can be very large which is very difficult to solve analytically? Here we assign some real-life problems of the TSP type and solve them analytically in this study.

2. Mathematical Formulation of TSP

TSP is a special kind of assignment problem in which a sales man may visit to a city only once which is explain by the mathematical notation

$$x_{ij} = \begin{cases} 1, & \text{if } i \text{ visits } j \\ 0, & \text{otherwise} \end{cases}$$

The following notation is used for the mathematical representation of the TSP.

d_{ij} = distance from city i to city j;

x_{ij}= amount of homogeneous product transported from source i and destination j.

Using the above notations, the TSP can be expressed in mathematical term as finding a set of x_{ij}'s, i=1, 2, … …, n; j=1, 2, … …, n to

$$\text{Optimize} \quad \sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij}x_{ij} \tag{1}$$

$$\text{Subject to} \quad \left.\begin{aligned} &\sum_{j=1}^{n} x_{ij} = 1, && i = 1,\cdots,n \\ &\sum_{i=1}^{n} x_{ij} = 1, && j = 1,\cdots,n \\ &x_{ij} = 0 \;\; or \;\; 1, && i = 1,\cdots,n, \quad j = 1,\cdots,n \end{aligned}\right\} \tag{2}$$

Associated to each traveling salesman problem there is a matrix called distance matrix [d_{ij}] where d_{ij} is the distance from city i to city j .

3. Distance Matrix

Let 1, 2,.......,n be the labels of the n cities and $d = d_{ij}$ be an $n \times n$ distance matrix where d_{ij} denotes the distance of traveling from city i to city j. Then general formation of the distance matrix is shown in Table 3.1.

Table 3.1 : Distance Matrix

	1	2	3	…	n	No. of visiting city
1	∞	d_{12}	d_{13}	…	d_{1n}	1
2	d_{21}	∞	d_{23}	…	d_{2n}	1
3	d_{31}	d_{32}	∞	…	d_{3n}	1
…	…	…	…	…	…	…
n	d_{n1}	d_{n2}	d_{n3}	…	∞	1
No. of visiting city	1	1	1	…	1	

If $d_{ij} = d_{ji}$, the problem is called *symmetric traveling salesman problem* (STSP).

4. Illustration of the Proposed Method

In this research, we compute a feasible optimization distance by an alternative way and for following this process, proposed method is named as "An Efficient Method for Solving Traveling Salesman Problem". Logical development of the proposed process for solving distance minimizing TSP is illustrated below:

Step-1: Construct the distance matrix from the given problem.

Step-2: Calculate the *Path Finding Indicator (PFI)* for each row as the sum of all the distance elements along every row and put them in front of the row on the right. In a similar fashion, calculate the PFI for each column and write them in the bottom of the distance matrix below corresponding columns.

Step-3: Select the highest PFI and observe the row or column along which this corresponds. If a tie occurs, select the PFI along which smallest distance elements appears.

Step-4: Make the assignment to the cell having smallest distance element corresponding to the highest PFI. If a tie occurs again, break it arbitrarily.

Step-5: No further consideration is required for the row and column along which the assignment is made.

Step-6: Calculate fresh PFI for the remaining sub-matrix as in Step 2 and assign following the procedure of Steps 3, 4 and 5. Continue the process until all the paths are selected.

Step-7: Complete the tour according to the selected location by visiting each location just the once. If it creates any sub-tour, go to Step 8.

Step-8: Eliminate the-sub tour in the following way-Choose the next smaller PFI successively to the highest PFI and make assignment following the procedure of Step 3 to Step 7. Continue the process until all the sub-tours are eliminated.

Then go to Step 9.

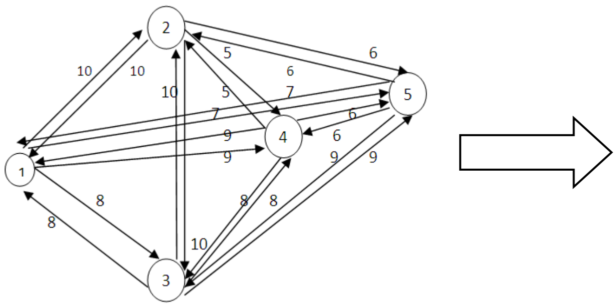
Step-9: Compute the total distance according to the selected tour.

5. Numerical Illustration

5.1 Example-1: A salesman has to visit five cities 1,2,3,4,5. He does not wish to visit any city twice before completing his tour of all cities and wants to return to the starting city by travelling a minimum distance.The distance from city1 to city2 is 10 km, city1 to city3 is 8 km, city1 to city4 is 9 km, city1 to city5 is 7 km. Find the optimal route and the minimum distance to be travelled by the salesman.

Step 1: According to the given problem construct the distance matrix.

Converted distance matrix



	1	2	3	4	5
1	-	10	8	9	7
2	10	-	10	5	6
3	8	10	-	8	9
4	9	5	8	-	6
5	7	6	9	6	-

Fig. 5.1.1: Distance assigned Network Problem 1 Table 5.1.1Symmetrical Matrix form

Step 2: Compute the sum of row and column distance elements.

Table 5.1.2 After Computing PFIs

	1	2	3	4	5	PFI
1	-	10	8	9	7	34
2	10	-	10	5	6	31
3	8	10	-	8	9	←35
4	9	5	8	-	6	28
5	7	6	9	6	-	28
PFI	34	31	35	28	28	

Step 3: Compute PFIs and comparing all PFIs select HPFI. Here HPFI is 35 which appears in the 3rd row & 3rd column. Select 3rd row arbitrarily. Along these HPFIs minimum distance 8 which appears in the cells (3, 1) & (3, 4). So arbitrarily allocate cell (2, 5). Delete 3rd row & 1st column.

Table 5.1.3 **After first selection**

	1	2	3	4	5	RPFI
1		**10**	**8**	**9**	**7**	**34**
2		**-**	**10**	**5**	**6**	**21**
3						
4		**5**	**8**	**-**	**6**	**19**
5		**6**	**9**	**6**	**-**	**21**
CPFI		**21**	**35↑**	**20**	**19**	

Again compute PFIs and comparing all PFIs select HPFI. Here HPFI is 35 which appears in the 3rd column. Along this HPFI minimum distance 8 which appears in the cells (1, 3) & (4, 3). For eliminate sub-tour, we choose (4,3) cell. Delete 4th row & 3rd column.

Step 4: Continue this proceed until to select all paths.

Table 5.1.4 After second selection

	1	2	3	4	5	RPFI
1		**10**		**9**	**7**	**←26**
2		**-**		**5**	**6**	**11**
3						
4						
5		**6**		**6**	**-**	**12**
CPFI		**16**		**20**	**13**	

Again compute PFIs and comparing all PFIs select HPFI. Here HPFI is 26 which appears in the 1st row. Along this HPFI minimum distance 7 which appears in the cell (1, 5). Delete 1st row & 5th column.

Table 5.1.5 After third selection

	1	2	3	4	5	RPFI
1						
2		**-**		**5**		**5**
3						
4						
5		**6**		**6**		**←12**
CPFI		**6**		**11**		

Again compute PFIs and comparing all PFIs select HPFI. Here HPFI is 12 which appears in the 5th row. Along this HPFI minimum distance 7 which appears in the cells (5, 2) & (5,4). For eliminate sub-tour, we choose (5, 2) cell. Delete 5th row & 2nd column.

Table 5.1.6 After fourth selection

	1	**2**	**3**	**4**	**5**	**RPFI**
1						
2		**-**		**5**		**5**
3						
4						
5						
CPFI				**5**		

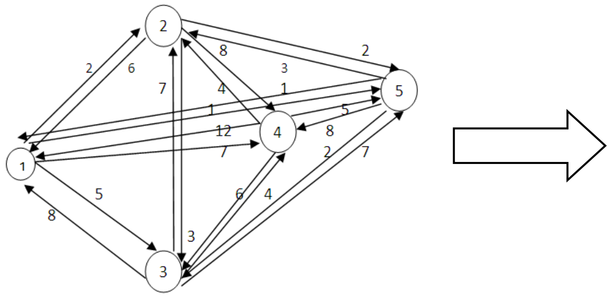
Step 5 : Selected paths : 3→1; 4→3; 1→5; 5→2; 2→4 ;

Tour : 3→1→5→2→4→3

Distance : 8 + 7 + 6 + 5 + 8 = 34 km.

5.2 Example -2: A salesman has to visit five cities 1,2,3,4,5. He does not wish to visit any city twice before completing his tour of all cities and wants to return to the starting city by travelling a minimum distance.The traveling distance from city1 to city2 is 2 km , city1 to city3 is 5 km , city1 to city4 is 7 km, city1 to city5 is 1 km; by alternate way city2 to city1 is 6 km, city2 to city3 is 3 km, city2 to city4 is 8 km, city 2 to city5 is 2 km; city3 to city1 is 8 km, city3 to city2 is 7 km , city3 to city4 is 4 km, city3 to city5 is 7 km; city4 to city1 is 12 km, city4 to city2 is 4 km , city4 to city3 is 6 km, city4 to city5 is 5 km; city5 to city1 is 1 km, city5 to city2 is 3 km , city5 to city3 is 2 km, city 5 to city 4 is 8 km. Find the optimal route and the minimum distance to be travelled by the salesman.

According to the given problem, we construct the following distance matrix.



Converted distance matrix

	1	2	3	4	5
1	**-**	**2**	**5**	**7**	**1**
2	**6**	**-**	**3**	**8**	**2**
3	**8**	**7**	**-**	**4**	**7**
4	**12**	**4**	**6**	**-**	**5**
5	**1**	**3**	**2**	**8**	**-**

Fig.5.2.1 Distance assigned Network Problem 2 Table 5.2.1 Asymmetrical Matrix form

Solving this problem by our Proposed Algorithm, we obtain

Selected paths: 3→4; 4→2; 2→5; 1→3; 5→1.

Tour: 3→4→2→5→1→3

Distance: 4 + 4 + 2 + 1 + 5 = 16.

6. Results and Discussion:

Table 6.1 shows a comparison among the solutions obtained by our proposed approach and the existing methods by means of the above two sample examples and it is seen that our proposed method gives better results.

Table 6.1 Comparison of results

Method	Feasible Solution		Number of Steps	
	Ex.-1	Ex.-2	Ex.-1	Ex.-2
Proposed Algorithm	**34**	**16**	**6**	**7**
Branch & Bounded Algorithm	**34**	**19**	**11**	**9**
Hungarian Method	**34**	**16**	**8**	**8**
Brute- Force Method	**34**	**16**	**12**	**12**

7. Conclusion

In this paper, we have proposed an algorithm for solving the traveling salesman problems occurring in real life situation. We also illustrate this algorithm numerically to test the efficiency of the proposed method. Comparative study among the solution obtained by the proposed method and the other existing methods by means of sample examples show that our proposed method provides effective result with less computational effort. Finally we may conclude that the proposed method may be considered as an alternative approach to solve TSP.

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